**Estimation of Income and price elasticities: An application of the Linear Expenditure System**

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**Introduction**

The International Labor Organization (ILO) plays an active role in collaborating with developing countries for the implementation of a 2030 agenda in the framework of the Sustainable Development Goals (SDGs), providing technical support to countries for the elaboration of sustainable development plans.

This note presents the methodological and technical details for the estimation of income and price elasticities, based on household survey data. These elasticities will constitute part of the background information that will be used for the simulation of macro-economic models that will guide policymaking and strengthen governments’ commitment to pursue the 2030 SDGs agenda.

The estimation of the mentioned income and price elasticities is based on the Linear Expenditure System (LES) methodology, as described by Stone(1954) and Frisch(1959). The main advantage of this methodology is to allow for the estimation of both income and price elasticities in absence to access to price information, relying on the properties of consumer theory.

The rest of this note is organized as follows. Section 1 presents a brief review of the methodological aspects that identify both price and income elasticities. Section 2 describes the implementation of the methodology and introduces the command “LES\_linear”, which can be used to estimate and report income and price elasticities. Section 3 describes the income and price elasticities for Tanzania, Ghana and Zimbabwe, as the chosen countries for the illustration of the methodologies. Section 4 concludes.

1. **Methodology: Linear expenditure system (LES)**

LES is a popular methodology used to estimate income and price elasticities based on cross-section data, when price data by sector is not available. While the identification of income elasticities does not rely on the assumptions imposed by consumer theory, the identification of price elasticities, in the absence of price information, relies entirely on the conditions imposed by consumer theory. Therefore, it is important to understand the sources of identification that lie behind this methodology.

The main assumption of the LES is that the utility function that individuals face, or some monotonic transformation of it, can be written as:

|  |  |
| --- | --- |
|  | (1) |

Where is the consumption of the good, and the parameters and are the parameters of the utility function. Following standard consumer theory, individuals attempt to maximize their utility subject to the budget constraint, where is the price of the good, and is total consumption.[[1]](#footnote-1) A solution for this maximization problem can be obtained using a Lagrangean system. In this case, the first-order conditions become:

|  |  |
| --- | --- |
|  | (2) |

where the parameter , Lagrange multiplier, represents the marginal utility of income. Imposing the assumption that the utility function is a homogeneous function of order one, we require . This implies that:

|  |  |
| --- | --- |
|  | (3) |

Substituting (3) into (2), we obtain the explicit demand for commodity j:

|  |  |
| --- | --- |
|  | (4) |

In this case, represent total expenditure on commodity , is often interpreted as the minimum level of expenditure on commodity , conditional on prices , and is interpreted as the total cost of a minimum commodities basket. represents the marginal allocation of resources for any excess resources above the cost of minimum commodities basket.

Based on equation (4), and assuming prices are fixed, income elasticity of demand for commodity can be estimated as follows:

|  |  |
| --- | --- |
|  | (5) |

Where is the total budget share of commodity . The two last expressions in equation (5) suggest at least two alternatives for the estimation of the parameters of interest: income elasticities conditional on , or the marginal budget share conditional on .

To estimate own and cross-price elasticities, we rewrite equation (4) and differentiate it with respect to prices. Taking into account that total expenditure is fixed, the own and cross elasticities will be given by:

|  |  |
| --- | --- |
| Cross price elasticity ||:  Own price elasticity ||: | (6) |

In other words, once estimates of demand income elasticity for all commodities () are obtained and the cost of minimum commodities basket () are identified, own and cross-price elasticities can be obtained. The next section discusses additional details regarding the identification of the utility parameters, income elasticity, and the cost of minimum commodities basket.

1. **Implementation**

The identification of most of the parameters corresponding to the LES is relatively straight forward. Adding an idiosyncratic error to equation (4), we can estimate the following system of equations:

If estimated jointly, given the constraints on the marginal budget shares, only of the parameters can be directly identified. However, independent regressions can also be used to identify all the marginal budget shares parameters.

A limiting factor of this approach is the identification of the parameters, or . These parameters can only be identified up to a constant. Without further restrictions, the estimated cost of minimum commodities basket, , is zero. Furthermore, for any given , corresponding values for can be obtained with .

Based on Taylor(1979), a common strategy to identify is using the Frisch parameter .[[2]](#footnote-2) This parameter is defined as the elasticity of the marginal utility of income with respect to income itself, and can be obtained by differentiating the Lagrange multiplier (equation 3) by income:

|  |  |
| --- | --- |
|  | (7) |

This parameter is often set at a value of , for most consumer groups, which implies . However, as Frisch(1959) suggests, this parameter should be expected to vary across the income distribution, with more negative values assigned to poorer households, and values closer to -1, for richer households.

Because own and cross-price elasticities depend on the estimation of and , and they are sensitive to any arbitrary choice of , will be defined based on its theoretical interpretation of being the cost of the minimum commodities basket. In other words, can be set at the national poverty or extreme poverty line, for the country of interest, to estimate all other relevant parameters. While this option works well for households across most of the income distribution, it may produce undesirable estimates when households fall below the poverty line. As a rule of thumb, if the poverty line is higher than 90% of the average income for the representative household, the Frisch parameter will be set at -10 (the lower limit discussed in Frisch (1959)), and will be identified based on the Frisch parameter (equation 6), otherwise, will be set to the poverty line.

To obtain accurate estimates for the parameters, the models use expenditure per equivalent adult, which are directly compared to the national poverty line.[[3]](#footnote-3) Furthermore, the log of household size, the share of household members at different age brackets, and dummies for geographical areas, are controlled for to account for other differences in consumption patterns not related to income differences. We also apply survey weights are applied to obtain estimates that better represent the behavior of the population. These factors are included in the model using a linear parametrization for .

The next section, I describe the command LES\_linear, which implements this methodology and reports all relevant information.

1. **Stata ado program: LES\_linear**

As previously described in the previous section, the estimation of income and price elasticities is a relatively simple task, but requires careful attention to when one is interested in estimating the income and price elasticities for specific subsamples. In order to facilitate the implementation of the LES methodology, I introduce the Stata command LES\_linear. This command has the following syntax:

LES\_linear [if], expenditures(varlist) poverty(#pline) [controls(varlist) wgt(varname)]

The sample of interest can be identified using the if qualifier. The final selected sample, however, is based on all non-missing data given the variables declared expenditures, controls, and wgt.

expenditures(varlist) is a required option and is used to specify all expenditures categories to be considered in the estimation of the income and price elasticities. Total expenditure is constructed as the sum of all expenditure variables specified in this option.

poverty(#pline) is a required option that provides the command with a scalar that contains the cost of the minimum commodities basket implied by the LES. As described in section 2, this minimum cost can be set equal to the poverty or extreme poverty line. If this reference value is larger than 90% of the average total expenditure, a Frisch parameter is used to identify the price elasticities.

controls(varlist) is a non-required option that can be used to specify other controls that would determine consumption patterns. For example, household size and household composition.

wgt(varname) is a non-required option that can be used to specify the weights that will be used to identify the expenditure models, and income and price elasticities.

After the command is executed, it stores in memory, two matrices, and three scalars

**scalars:**

r(avg\_exp) Average weighted expenditure for the selected sample

r(gamma) Cost of the Minimum commodities basket Bundle

r(frisch) Implicit Frisch parameter

**matrices:**

r(nij) Matrix of own and cross-price elasticities

r(all) Matrix of marginal budget share, average budget share, and income elasticities.

1. **Data and Results**

This section, I illustrate the application of the LES\_linear command to estimate the income and price elasticities for various sectors for the countries of Tanzania, Ghana and Zimbabwe, based on the official consumption aggregates. Data from the 2012 Household budget survey in Tanzania, the 2017 Ghana-Living Standards Survey, and the 2010 Living Conditions Monitoring Survey for Zimbabwe are used.

To obtain results that are comparable across countries, all estimations are obtained using survey weights, and expenditures, measured in local currency are normalized using adult equivalence scales. Minor trimming of the data was done. Any record with zero total expenditure is excluded from the sample, and for Tanzania, the record with the highest total expenditure and expenditure in education are also excluded from the analysis. For all countries, dummies for region (urban-rural), the log of household size, and shares of individuals in different age brackets are also included. Summary statistics on these variables are provided in table 1.

Table 1 Summary Statistics

|  |  |  |  |
| --- | --- | --- | --- |
|  | Tanzania (2012) | Ghana (2017) | Zimbabwe (2010) |
| Total Expenditure per Eq Adult | 62,855 | 4,078 | 188,808 |
| Gini | 0.357 | 0.421 | 0.548 |
| Headcount Poverty | 29.9% | 24.9% | 65.0% |
| Rural-Urban |  |  |  |
| Urban | 18.7% | 50.7% | 34.7% |
| Rural | 71.2% | 49.3% | 65.3% |
| Dar-es-Salam | 10.1% |  |  |
| Household Characteristics |  |  |  |
| Log (household size) | 1.784 | 1.561 | 1.780 |
| Share Children 0-5 | 19.9% | 15.4% | 16.3% |
| Share Children 6-15 | 26.1% | 25.1% | 30.3% |
| Share Adults 16 -24 | 16.5% | 16.8% | 19.4% |
| Share Women 25 -64 | 17.5% | 20.4% | 16.2% |
| Share Men 25 -64 | 15.9% | 17.3% | 15.4% |
| Share Elderly 65+ | 4.1% | 5.0% | 2.4% |
| LES parameters |  |  |  |
| Implicit Frisch parameters | -2.383 | -1.760 | -4.411 |
| Cost of Minimum Basket | 36,482 | 1,761 | 146,009 |

Note: All statistics are estimated using survey weights. For Tanzania and Zimbabwe, extreme values for total expenditure and expenditure on education were excluded. Expenditure per capita and cost of Minimum Basket are expressed in local currency.

As can be seen in Table 1, Zimbabwe is the country with the lowest economic performance among the countries used for this illustration. The estimates indicate that about 65% of the population lives below the poverty line, with a Gini coefficient of 0.548. Both Ghana and Tanzania have better indicators, with headcount poverty rates of only 29.9% (Tanzania) and 24.9% (Ghana). While Ghana has the lowest headcount poverty rate, their inequality (Gini 0.421) in 2017 is higher than that observed in Tanzania (Gini 0.357).

In terms of geographical characteristics, Zimbabwe and Tanzania have a larger segment of their population living in rural areas. The (geometrical) average household size in Tanzania and Zimbabwe is 5.9, whereas Ghana it is 4.7. Based on the average household composition, Zimbabwe has fewer individuals living in households with elderly household member, but a larger proportion of households with children and young adults.

Table 2 Budget Shares, Marginal Budget Shares and Income Elasticities

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | Tanzania | | | Ghana | | | Zimbabwe | | |
| Expenditures | Marginal Budget Share | Budget Share | Income Elasticity | Marginal Budget Share | Budget Share | Income Elasticity | Marginal Budget Share | Budget Share | Income Elasticity |
| Food | 0.2175 | 0.6319 | 0.3442 | 0.2378 | 0.5016 | 0.4742 | 0.4681 | 0.6683 | 0.7005 |
| Alcoholic beverages, tobacco, Narcotics | 0.0019 | 0.0053 | 0.3633 | 0.0033 | 0.0071 | 0.4639 | 0.0092 | 0.0136 | 0.6717 |
| Clothing and Footwear | 0.0479 | 0.0685 | 0.6994 | 0.0532 | 0.0901 | 0.5904 | 0.0169 | 0.0353 | 0.4779 |
| Housing | 0.0794 | 0.1307 | 0.6076 | 0.0980 | 0.0950 | 1.0312 | 0.1484 | 0.0975 | 1.5221 |
| Furnishing | 0.0591 | 0.0146 | 4.0488 | 0.0447 | 0.0427 | 1.0481 |  |  |  |
| Health Expenditure | 0.0423 | 0.0254 | 1.6624 | 0.0050 | 0.0077 | 0.6523 | 0.0041 | 0.0033 | 1.2520 |
| Transportation | 0.2686 | 0.0445 | 6.0427 | 0.3239 | 0.0697 | 4.6454 | 0.0878 | 0.0347 | 2.5326 |
| Communication | 0.0772 | 0.0443 | 1.7419 | 0.0304 | 0.0247 | 1.2306 | 0.0245 | 0.0229 | 1.0675 |
| Recreation and Entertainment | 0.1083 | 0.0130 | 8.3351 | 0.0201 | 0.0137 | 1.4741 | 0.0044 | 0.0009 | 5.0326 |
| Education | 0.0413 | 0.0100 | 4.1429 | 0.1496 | 0.1235 | 1.2109 | 0.1044 | 0.0543 | 1.9226 |
| Restaurants and Hotels | 0.0238 | 0.0028 | 8.4984 | 0.0026 | 0.0003 | 9.6833 |  |  |  |
| Miscellaneous | 0.0328 | 0.0091 | 3.5917 | 0.0313 | 0.0240 | 1.3047 | 0.1324 | 0.0693 | 1.9105 |

Table 3 Cross and Own Price Elasticities: Tanzania

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  | Price | | | | | | | | | | | |
|  |  | Food | Alco | Clot | Hous | Furn | Heal | Tran | Comm | Recr | Educ | Rest | Misc |
| Commodities | Food | **-0.331** | -0.002 | -0.017 | -0.034 | 0.004 | -0.003 | 0.023 | -0.004 | 0.011 | 0.003 | 0.002 | 0.002 |
| Alco | -0.196 | **-0.154** | -0.018 | -0.035 | 0.004 | -0.003 | 0.025 | -0.004 | 0.012 | 0.003 | 0.003 | 0.002 |
| Clot | -0.378 | -0.003 | **-0.327** | -0.068 | 0.007 | -0.005 | 0.048 | -0.008 | 0.023 | 0.005 | 0.005 | 0.003 |
| Hous | -0.328 | -0.003 | -0.029 | **-0.314** | 0.006 | -0.005 | 0.041 | -0.007 | 0.020 | 0.004 | 0.004 | 0.003 |
| Furn | -2.189 | -0.018 | -0.196 | -0.394 | **-1.658** | -0.031 | 0.276 | -0.048 | 0.131 | 0.030 | 0.029 | 0.019 |
| Heal | -0.899 | -0.007 | -0.080 | -0.162 | 0.017 | **-0.710** | 0.113 | -0.020 | 0.054 | 0.012 | 0.012 | 0.008 |
| Tran | -3.267 | -0.027 | -0.292 | -0.589 | 0.062 | -0.046 | **-2.123** | -0.072 | 0.196 | 0.044 | 0.043 | 0.028 |
| Comm | -0.942 | -0.008 | -0.084 | -0.170 | 0.018 | -0.013 | 0.119 | **-0.752** | 0.057 | 0.013 | 0.013 | 0.008 |
| Recr | -4.506 | -0.038 | -0.403 | -0.812 | 0.085 | -0.064 | 0.569 | -0.099 | **-3.227** | 0.061 | 0.060 | 0.039 |
| Educ | -2.240 | -0.019 | -0.200 | -0.403 | 0.042 | -0.032 | 0.283 | -0.049 | 0.134 | **-1.708** | 0.030 | 0.019 |
| Rest | -4.594 | -0.038 | -0.411 | -0.828 | 0.087 | -0.065 | 0.580 | -0.101 | 0.276 | 0.063 | **-3.505** | 0.039 |
| Misc | -1.942 | -0.016 | -0.174 | -0.350 | 0.037 | -0.028 | 0.245 | -0.043 | 0.117 | 0.026 | 0.026 | **-1.490** |

Table 4 Table 3 Cross and Own Price Elasticities: Ghana

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  | Price | | | | | | | | | | | |
|  |  | Food | Alco | Clot | Hous | Furn | Heal | Tran | Comm | Recr | Educ | Rest | Misc |
| Commodities | Food | **-0.443** | -0.002 | -0.028 | -0.019 | -0.008 | -0.002 | 0.054 | -0.004 | -0.001 | -0.018 | 0.001 | -0.003 |
| Alco | -0.170 | **-0.266** | -0.028 | -0.018 | -0.008 | -0.002 | 0.053 | -0.003 | -0.001 | -0.018 | 0.001 | -0.003 |
| Clot | -0.216 | -0.003 | **-0.371** | -0.023 | -0.010 | -0.003 | 0.068 | -0.004 | -0.001 | -0.023 | 0.001 | -0.004 |
| Hous | -0.378 | -0.005 | -0.062 | **-0.627** | -0.018 | -0.005 | 0.118 | -0.008 | -0.002 | -0.040 | 0.001 | -0.006 |
| Furn | -0.384 | -0.005 | -0.063 | -0.041 | **-0.614** | -0.005 | 0.120 | -0.008 | -0.002 | -0.040 | 0.001 | -0.007 |
| Heal | -0.239 | -0.003 | -0.039 | -0.026 | -0.011 | **-0.374** | 0.075 | -0.005 | -0.001 | -0.025 | 0.001 | -0.004 |
| Tran | -1.702 | -0.024 | -0.278 | -0.183 | -0.080 | -0.023 | **-2.109** | -0.034 | -0.010 | -0.179 | 0.006 | -0.029 |
| Comm | -0.451 | -0.006 | -0.074 | -0.048 | -0.021 | -0.006 | 0.141 | **-0.708** | -0.003 | -0.047 | 0.002 | -0.008 |
| Recr | -0.540 | -0.008 | -0.088 | -0.058 | -0.025 | -0.007 | 0.169 | -0.011 | **-0.841** | -0.057 | 0.002 | -0.009 |
| Educ | -0.444 | -0.006 | -0.073 | -0.048 | -0.021 | -0.006 | 0.138 | -0.009 | -0.003 | **-0.735** | 0.001 | -0.008 |
| Rest | -3.548 | -0.051 | -0.580 | -0.381 | -0.167 | -0.047 | 1.107 | -0.072 | -0.021 | -0.373 | **-5.491** | -0.060 |
| Misc | -0.478 | -0.007 | -0.078 | -0.051 | -0.023 | -0.006 | 0.149 | -0.010 | -0.003 | -0.050 | 0.002 | **-0.749** |

Table 5 Cross and Own Price Elasticities: Zimbabwe

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  | Price | | | | | | | | | | | |
|  |  | Food | Alco | Clot | Hous | Furn | Heal | Tran | Comm | Recr | Educ | Rest | Misc |
| Commodities | Food | **-0.553** | -0.008 | -0.022 | -0.045 |  | -0.002 | -0.010 | -0.012 | 0.000 | -0.021 |  | -0.028 |
| Alco | -0.378 | **-0.160** | -0.021 | -0.043 |  | -0.002 | -0.010 | -0.012 | 0.000 | -0.021 |  | -0.026 |
| Clot | -0.269 | -0.006 | **-0.123** | -0.031 |  | -0.001 | -0.007 | -0.008 | 0.000 | -0.015 |  | -0.019 |
| Hous | -0.856 | -0.018 | -0.048 | **-0.442** |  | -0.004 | -0.022 | -0.026 | 0.000 | -0.047 |  | -0.060 |
| Furn |  |  |  |  |  |  |  |  |  |  |  |  |
| Heal | -0.704 | -0.014 | -0.039 | -0.080 |  | **-0.287** | -0.018 | -0.022 | 0.000 | -0.038 |  | -0.049 |
| Tran | -1.424 | -0.029 | -0.080 | -0.162 |  | -0.006 | **-0.611** | -0.044 | 0.000 | -0.078 |  | -0.099 |
| Comm | -0.600 | -0.012 | -0.034 | -0.068 |  | -0.003 | -0.016 | **-0.261** | 0.000 | -0.033 |  | -0.042 |
| Recr | -2.829 | -0.058 | -0.158 | -0.321 |  | -0.012 | -0.074 | -0.087 | **-1.140** | -0.154 |  | -0.198 |
| Educ | -1.081 | -0.022 | -0.060 | -0.123 |  | -0.005 | -0.028 | -0.033 | 0.000 | **-0.495** |  | -0.076 |
| Rest |  |  |  |  |  |  |  |  |  |  |  |  |
| Misc | -1.074 | -0.022 | -0.060 | -0.122 |  | -0.004 | -0.028 | -0.033 | 0.000 | -0.059 |  | **-0.508** |

Table 2 provides estimations of the marginal budget share, budget share and income elasticities for 12 sectors in Tanzania and Ghana, and 10 sectors in Zimbabwe, based on the official consumption aggregates included in all consumption surveys. As expected, food expenditure is the most important commodity across all countries. In Ghana, the budget share is the lowest, at 50.2%, whereas Tanzania and Zimbabwe budget share is 63.2% and 66.8%, respectively. Perhaps more interesting is that income elasticity in Tanzania is the lowest (0.34), whereas in Zimbabwe is twice as high at 0.7. Other than food, only Alcohol and Clothing appear to be inelastic with respect to income for all three countries. In Tanzania, expenditure on recreation, restaurants and hotels, transportation, education, and furnishing present income elasticities above 4. For Ghana, only expenditure on restaurants and hotels, and transportation present similar elasticities, while expenditure on recreation and entertainment presents equally large income elasticities in Zimbabwe.

Tables 3 through 5 provide the own and cross-price elasticities for each country. Most of the cross-price elasticities are negative, which suggests all commodities are complements. There are, however a few exceptions. In Tanzania, the cross-price elasticities of all goods with respect to prices of furnishing, transportation, recreation, education, restaurants and hotels, and miscellaneous expenditures, are positive, which suggests they are substitutes. These elasticities are, however, quite small. Similar patterns are observed in Ghana (with respect to transportation and, restaurants and hotels), and Zimbabwe (recreation).

1. **Conclusions**

This note I describe the Linear Expenditure system methodology for the estimation of income and price elasticities based on household survey data, which can be applied when information on prices is not readily available. I also introduced a new Stata command LSE\_linear, which implements the LES methodology, and constructs all the elasticities of interest. This command requires data on aggregated expenditures (by commodities), and an appropriate poverty line, which represents the cost of the minimum commodity basket individuals require. This new command will facilitate the estimation of income and price elasticities for specific population groups, which can be used as input for other simulation models.

1. **References**

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Stone, R. (1954) Linear Expenditure Systems and Demand Analysis: An Application to the Pattern of British Demand. *The Economic Journal,* 64(255), 511-527.

Taylor, L. (1979) Macro Models for Developing Countries. New York: McGraw-Hill.

1. The LES strategy assumes all available income is fully consumed across all types of goods. This implies income elasticity of total consumption is 1. This also implies that there are no explicit savings in this method, unless it is included as an additional type of good consumers choose from. [↑](#footnote-ref-1)
2. Frisch(1959) introduces this parameter as a measure of consumer welfare, but it can also be used to identify the cost of the minimum commodities basket, for the LES. [↑](#footnote-ref-2)
3. For simplicity, we use the equivalence scales provided on each survey. These are meant to adjust for differences in consumption needs between children and adults. [↑](#footnote-ref-3)